Eigenvalues and eigenvectors of macroeconometric models

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can also observe whether there are complex valued eigenvalues (and consequently cyclical movements). In this context we notice that macroeconometric models often incorporate one or the other form of an accelerator mechanism. See Muet (1979).

Finally, we note that in a number of studies the cyclical behaviour of macroeconometric models is investigated by using methods of the so-called spectral analysis. For a review of these methods see Wolters (1980). In the spectral analysis the emphasis lies on the study of the impact of the stochastic disturbances in econometric models. As said, however, this falls outside the scope of the present study.

1.3 Object and summary of this study

Above we have argued that while analysing the dynamic behaviour of a theoretical macroeconomic model, one tries to relate the properties of the eigenvalues pertaining to the model to the values of the model coefficients. Due to the relatively simple structure of many of these models this can often be done 'straightforwardly'.

As regards to the macroeconometric models the situation is more complex. Beforehand it is often unclear what will be the characteristics of the set of eigenvalues associated with a model. However, insight into the principal determinants of the eigenvalues, especially of the dominant eigenvalue, is important since the dynamic properties of a model depend on these eigenvalues.

So far, we have limited our discussion to the eigenvalues of a model; i.e. we have not considered yet the eigenvectors that pertain to the eigenvalues. However, we mention here that, while examining the dynamic properties of an econometric model, one can decompose the matrix of which the eigenvalues have to be analysed, by means of the so-called spectral decomposition. The eigenvectors of the matrix play an important role in this decomposition. In the economic literature not much attention is given to the characteristics and the meaning of these eigenvectors. An exception is the study of Borghers and Plasmans (1970), in which, by using methods of factor analysis, the eigenvectors are related to the time series of the variables of the model.

In the light of our above discussion, we formulate the following theme of our investigations: *The object of this study is twofold. First, we investigate and develop methods that can be used in order to find characteristics and an interpretation in economic theoretical terms of the eigenvalues and eigenvectors that are associated with linear macroeconometric models. Special attention will be given to the stability properties of such models. Second, we apply these methods to a number of typical (Keynesian) macroeconometric models. In particular, we analyse the Grecon model, a model describing the Dutch post-war economy.*

Now, we will briefly summarize the contents of the chapters that follow. In chapter 2 we first treat some basics; in particular, the general form of a dynamic linear simultaneous equation model, the eigenvalue problem of a matrix, the spectral decomposition and the Jordan decomposition. In the last section of this chapter we examine some practical problems that can arise in the study of the eigensystem of linear models: to mention, the question whether one can determine for a given model the exact number of eigenvalues that are equal to zero. We point out a way to approach the latter question, and resolve, by using this approach, a discussion that arose some years ago in the literature about a specific macroeconometric model.

A method that is useful in order to obtain insight into the relation of, on the one side, an eigenvalue associated with a model and, on the other side, the coefficients of the model, is the so-called sensitivity analysis of the eigenvalue with respect to (relatively) small changes in the coefficients of the model. This method is studied in chapter 3. It is the purpose of the sensitivity analysis to identify those coefficients of the model that have a (relatively) large impact on specific eigenvalues. By this we mean that a (relatively) small change in the value of a coefficient produces a (relatively) large change in the value of an eigenvalue under consideration. The reason for the success of this method lies in the fact that, in practice, it often appears that only a limited number of the coefficients of a model have a substantial impact on a given eigenvalue. In the literature, while studying an eigenvalue of interest, it is usage to inspect the impact of the model coefficients one by one. However, we will argue that it has advantages if one inspects the effect on the eigenvalue of a simultaneous change in sets of deliberately chosen model coefficients. Furthermore, we demonstrate that our approach unifies a number of procedures that are discussed in the literature. Finally, we show that the sensitivity analysis of the elements of the eigenvectors associated with a model can proceed along similar lines.

In an empirical study Uebe and Fischer (1974) conclude that the value of the dominant eigenvalue (i.e. the eigenvalue having the largest absolute value) of a large number of macroeconometric models does not change appreciably if the values of small coefficients of the model are put equal to zero. In this sense Uebe and Fischer call the dominant eigenvalue robust. They did not succeed in finding an explanation of the observed feature. In chapter 4, we demonstrate that the occurrence of a specific type of model equation in the relevant models should be held responsible for the observed robustness. Moreover, we argue that as a consequence of the relevant equations the dominant eigenvalues are approximately equal to unity, which is, as known, the critical value as regards to the stability or instability of a model.

The eigenvalues associated with a linear model are in fact the eigenvalues of a reduced form matrix of the model. As said above, in the literature one can find conditions that can be used to determine whether a model is stable without actually computing the eigenvalues. We notice in chapter 5 that these conditions are usually presented in terms of properties of the reduced form matrix itself. However, an econometric model is always specified and constructed in the structural form. Therefore, in order to facilitate interpretation of the stability or instability of a model in terms of the equations and coefficients in which it is originally given, we study in chapter 5 conditions that relate stability of a model to properties of the structural form matrices. We first analyse the stability of models of which the structural form matrices are nonnegative. An intuitively appealing interpretation of the stability of such models is presented in terms of the size of the coefficients in the associated structural form equations. Next, we investigate in a similar way the models of which the structural form matrices possess positive as well as negative elements. For a number of examples of macroeconomic models we show that the negative structural form coefficients in these models have a stabilizing impact. At first, the analysis of the examples is carried out by examining exhaustively all possible combinations of values of the model coefficients; i.e. no explicit attention is given to the structure of the matrices involved in relation to the results obtained. Next, motivated by our findings we examine, in general matrix terms, the impact of the negative elements of a matrix on its eigenvalues. We present some theorems and subsequently use these theorems in order to point out the relevant common structure in the matrices involved in the mentioned examples.

The eigenvectors of the linear model are investigated in chapter 6. Under certain conditions the vector of the endogenous variables of a model can be expressed as a linear combination of the complete set of eigenvectors. This is done by using the spectral decomposition to the final form that can be associated with a model. We analyse systematically the components that play a role in this decomposition. Next, we describe a method that can be used in order to identify the key eigenvectors out of the complete set of eigenvectors. Our procedures are illustrated with an application to a small (Keynesian) macroeconometric model.

In chapter 7 we apply the results that are obtained in the former chapters to the Grecon model. We first present the dominant eigenvalues of 11 consecutive versions of this model. It appears that there is a variation in the size of the dominant eigenvalues. Furthermore, the dominant eigenvalues of the earlier versions are real valued, whereas those of the more recent versions are complex valued. In chapter 7 we choose as a reference version the model Grecon 81 – A. We examine in detail the eigenvalues associated with this model. It follows from our investigations that the Grecon model 81 – A can be divided into two submodels. Each of the submodels corresponds to an important eigenvalue of the model (among them is the dominant eigenvalue). By analysing the submodels we are able to identify the main determinants of these eigenvalues, and interpret them economically in terms of the structural form coefficients of the model. Next, given the analysis of the Grecon model 81 – A, we can briefly examine the spectral radii of the other versions of the Grecon model. We also study the eigenvectors associated with the Grecon model 81–A. Specific elements of the eigenvectors are related to the submodels mentioned above.

Finally, we conclude in chapter 8, in the light of the previous chapters, with some general observations.